

Stability of Predictive Control in Job Shop System with Reconfigurable Machine Tools for Capacity Adjustment

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ABSTRACT

Due to changes in individual demand, manufacturing processes have become more complex and dynamic. To cope with respective fluctuations as well as machine breakdowns, capacity adjustment is one of the major effective measures. Instead of labor-oriented methods, we propose a machinery-based approach utilizing the new type of reconfigurable machine tools for adjusting capacities within a job shop system. To economically maintain desired work in process levels for all workstations, we impose a model predictive control scheme. For this method we show stability of the closed-loop for any feasible initial state of the job shop system using a terminal condition argument. For a practical application, this reduces the computation of a suitable prediction horizon to controllability of the initial state. To illustrate the effectiveness and plug-and-play availability of the proposed method, we analyze a numerical simulation of a four workstation job shop system and compare it to a state-of-the-art method.

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BIBA – Bremer Institut für Produktion und Logistik GmbH at the University of Bremen, Hochschulring 20, 28359 Bremen, Germany University of Bremen, Research Cluster LogDynamics and Faculty of Production Engineering, Badgasteiner Straße 1, 28359 Bremen, Germany This article is the extension of a conference paper entitled "Predictive Control of a Job Shop System with RMTs using Equilibrium Terminal Constraints" presented at the 6th International Conference on Dynamics in Logistics (LDIC2018).

Keywords: Reconfigurable machine tool · Capacity adjustment · Model predictive control · Stability

1 INTRODUCTION

Nowadays, consumers demand individualized products in small quantities with short delivery time. Consequently, manufacturers are confronted with the challenge to react to demand and market fluctuations quickly, efficiently and effectively. This tendency renders manufacturing processes to be more complex and dynamic. In general, such processes are subject to external disturbances, e.g. rush orders, and internal factors, e.g. machine breakdown or intended adjustments by system design. The current manufacturing paradigms, which aim at producing products at low cost and high qualities, cannot deal with this requirement in a satisfactory manner. To deal with the resulting performance degradation and to achieve a good shop floor performance, capacity adjustment is one of the major effective tools [1]. Here, even small modifications during a high load period may improve performance significantly [2].

Typically, capacity adjustment is done by purchasing new equipment, employing temporary workers, extending working times and so forth. These options offer flexibility, but are not long-term sustainable and expensive especially in the western countries where the labor cost is high [3]. In the perspective of sustainability, reconfigurable machine systems (RMS) may fill this gap. Similar to the mentioned alternatives, these are flexible by construction but also allow to adapt capacity and functionality within a certain range. The key characteristics of RMS include modularity, scalability, convertibility, customization, and diagnosability [4]. Such systems show significant impact on sustainable manufacturing to improve the responsiveness to market changes while remaining cost-effective [5]. On the downside, the resulting planning problem is of mixed integer nature, which calls for new methods to costeffectively utilize flexibility, capacity scalability and functionality of RMS in the dynamic manufacturing systems [6].

The main essential component rendering RMS successful is the reconfigurable machine tool (RMT). These machine tools are modularly designed for a customized range of operation requirements, combining the advantages of high productivity of dedicated machine tools (DMT) with high flexibility of flexible machine tools (FMT). Also, these tools are designed in accordance with the concept of sustainability, such as improving flexibility, shorting delivery times, reducing material consumptions, and enhancing responsiveness in the presence of demand fluctuation [7].

RMTs may be used to balance capacities and loads. Yet, such an adaptation requires a short reconfiguration time to adjust capacity and functionality [8]. Relying on production capability of RMTs, the authors studied a single product line to satisfy demand changes. The production capacity was increased or decreased through adding or removing auxiliary modules for performing different operations. In [5], purchasing new RMTs was used to increase capacity while minimizing reconfiguration cost and capital investment cost. To exploit the best configuration, a mixed integer linear programming problem was formulated. Two cases concerning cost management were presented to demonstrate the efficiency of the proposed method. Taking into account frequent reconfiguration cost, the responsiveness of RMT was measured and evaluated by operational capability and machine reconfigurability metrics [9]. This reconfigurability and flexibility can be exploited best within manufacturing systems with high product diversity at small lot sizes, which is the case, e.g., for job shop systems [1, 10]. In [11], the recent development of RMTs was studied and results indicated that the contributions were mainly derived from configuration optimization, architecture design and system integration and control.

However, in the above literature, RMTs are only the source and enabler in terms of planning. To effectively utilize these tools on the operational level, we require control methods incorporating the dynamic characteristics of RMTs and disturbances of the process. As job shop systems offer high variability, researchers focused on this type of manufacturing system and studied the impact of RMTs and respective control methods on performance measures of such systems. Since job shop systems may suffer from high work in process (WIP) levels and therefore unreliable due dates and long lead times within a production network [12], they have been mostly studied using discrete event simulation (DES). However, this approach requires a rather high modeling effort and is limited in the time frame. On the other hand, a continuous time modeling and simulation method may provide an additional research possibility on manufacturing process control [13, 14, 15]. This method has been evaluated via a state space model setting and further compared with DES. The results indicated that there was a subtle difference in terms of mean and variation of WIP and lead time [16]. Independent from the approach, stability of the closed-loop is of utmost importance. Here, process stability refers to performance indicators (e.g. WIP) remaining bounded as converging to desired values or an acceptable stability region. In [17], a comparison regarding analysis of stability regions was conducted from both perspectives (macroscopic and microscopic), i.e. continuous modeling by mathematical theory and simulation results from DES. The authors indicated that such an approximation made by a mathematical model is suitable and effective for stability analysis. This method allows to determine control parameters to ensure stability of a production network faster than a repeated trial and error approach. However, only steady state stability was discussed, the tracking control problem was not taken into account.

In this paper, we consider job shop systems and first follow the approach from [14] to directly control the WIP level by adapting the number of RMTs within the workstations separately. To balance capacities and loads in the system, we then impose a model predictive control (MPC) scheme, which is widely applied for mechanical or chemical systems [18], inventory management in supply chain [19] and has grown mature over the last decades [20]. As the method allows dealing with constraints explicitly while studying an finite horizon optimization problem iteratively and being inherently robust, it is readily applicable to assign RMTs and achieve a good shop floor performance in the presence of demand fluctuations. For this method, we show stability of the MPC closed-loop system by imposing equilibrium terminal conditions. Moreover, we illustrate the effectiveness of our approach by a numerical simulation subject to a range of order release rates and limited available capacity.

The remainder of this paper is organized as follows: The problem definition is given in Section 2. Thereafter, the basic MPC algorithm with equilibrium terminal conditions will be introduced in Section 3. In Section 4, an illustrative example of a job shop system with RMTs and DMTs is investigated and simulation results are presented. Last, conclusion and future research directions are presented in Section 5.

Notation: Throughout this work we denote the natural numbers including zero by \mathbb{N}_0 and the nonnegative reals by $\mathbb{R}_{\geq 0}$. The Euclidean norm is denoted by $\|\cdot\|$. For any vector $x \in \mathbb{R}^n$, $n \in \mathbb{N}$, $\|x\|_2 = \sqrt{\sum_{j=1}^p x_j^2}$ represents the 2-norm. The 2-norm of matrix $A \in \mathbb{R}^{n \times n}$ is denoted as $\|A\|_2 = \sqrt{\lambda_{max}(A^\top A)}$, where λ_{max} is the maximum

Publications	Contributions	Methodology Transfer function and proportional controller State space and proportional controller	
JH. Kim et.al. [14]	Present a dynamic multi-workstation model with closed-loop capacity control include disturbance		
N. Duffie et.al. [16]	Build up a discrete model of production network with local capacity control and compared with DES		
B. Scholz-Reiter et.al. [23]	Analyze dynamic behavior and performance of capacity control of production network via Vensim DSS software	Bio-inspired	
H.R. Karimi et.al [24]	Investigate a class of production network for capacity changes with time-delay and show the stability	H_{∞} control	
I.K. Sagawa and M.S. Nagano [25]	Present a model of multi-product job shop system to maintain a desired WIP level	Bond graph proportional controller	

Table 1: WIP control by means of control theory

eigenvalue of the matrix A, $||Ax||_2 \leq ||A||_2 ||x||_2$. Furthermore, we call a continuous function $\gamma: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ of class \mathcal{K}_{∞} if it is zero at zero, strictly increasing and unbounded. Similarly,a continuous function $\beta: \mathbb{R}_{\geq 0} \times \mathbb{N}_0 \to \mathbb{R}_{\geq 0}$ is said to be of class \mathcal{KL} if for each $n \geq 0$ it satisfies $\beta(\cdot, n) \in \mathcal{K}_{\infty}$ and for each r > 0 it is strictly decreasing in its second argument with $\lim_{n\to\infty} \beta(r, n) = 0$.

2 PROBLEM DEFINITION

Job shop manufacturing systems provide high flexibility in conjunction with cross-link information and multi-directional flow, which is indispensable for high customization with low repetition rates. These properties are beneficial for often changing products but may lead to bottlenecks in one or multiple machines or workstations. Because it may contain reentrant lines to complete products in the process, the orders may return to the same machine many times to perform different steps of the process. Meanwhile, some machines or workstations may lay idle. The resulting bottlenecks, in turn, will cause high work in process (WIP), long lead time, low machine utilization, and low due date reliability for the overall system [1]. Generally, the decisions for planning and control can be classified into three categories: strategic, tactical and operational. Here, we specifically focus on the operational layer, which considers shortterm decisions and is related to optimally controlling the manufacturing process. In particular, we assume that the sequence of orders to be processed is fixed.

In order to shorten lead time and improve the reliability of delivery time, one could release orders earlier, which is intended to increase output rates. Doing so may destabilize the system, cause unbounded growth of WIP, additional inventory cost, requirement of large storage space and even loss of consumers [21]. Since the WIP level is essential for all key performance indicators [22], we propose to control the WIP level

directly through utilizing capacity adjustments to eliminate or periodically shift bottlenecks within the process. An overview on typical investigations on the control of WIP is given in Tab. 1.

The mentioned works significantly improved the control performance. However, they mainly focused on labor-oriented approaches. Here, we consider machinery-based capacity adjustment via RMTs, i.e. we adapt the number of RMTs assigned to specific tasks on the shop floor. Hence, our aim is to design a feedback to allocate the RMTs within the job shop such that a certain WIP level is tracked and then show the stability of the controlled system. Within [15], the authors modeled a job shop system with RMTs and decomposed it into two operators for the design of robust stabilizing controllers. Afterwards, they designed a PI tracking controller with respect to WIP. The tracking performance could be ensured even in the presence of bounded uncertainty by robust right coprime factorization. Yet, the feedback cannot handle constraints explicitly and effectively.

Within this paper, we consider a simple flow model of a job shop system with p workstations. The job shop is given by a fully connected graph $\mathcal{G} = (V, P)$, where the set of vertexes $V := \{1, \ldots, p\}$ represents the workstations and

$$P := \begin{bmatrix} p_{11} \ p_{12} \cdots \ p_{1p} \\ p_{21} \ \ddots \ \cdots \ p_{2p} \\ \vdots \ \vdots \ \ddots \ \vdots \\ p_{p1} \ p_{p2} \cdots \ p_{pp} \end{bmatrix}$$

the flow probability matrix between the workstations. As shown in Figure 1, $i_{1j} \cdots i_{pj}$ represent the input rates of each workstation to workstation *j*, where i_{0j} denotes the order release rate to workstation *j*. Moreover, $o_{j0}, o_{j1} \cdots o_{jp}$ represent the respective output rates and o_{j0} the output rate of final products of workstation *j*. We like to note that this procedure requires that knowledge of flow of products is digitalized and not expert knowledge of workers on the shop floor.

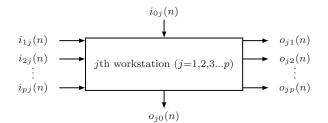


Fig. 1: jth workstation in multi-workstation production system

To avoid order-machine specialties – e.g. certain manufacturing steps for a product can be executed on one machine only – we suppose that each of the workstations features n^{DMT} identical DMTs, which may operate with production rate $[0, r^{\text{DMT}}]$. The number of RMTs is controlled by our input variable u and each RMT may operate with production rate $[0, r^{\text{RMT}}]$. The work in process level WIP_j is defined as the number of orders waiting to be processed at workstation j, hence its rate of change is given by the difference between rates of input and output orders I and O. Based on Fig. 1, we obtain

$$I_j\!:=\sum_{l=0}^p i_{lj} \quad \text{and} \quad O_j\!:=\sum_{l=0}^p o_{jl}$$

Therefore, time dependent dynamics of WIP_j can be computed via its previous value, the inputs from other workstations, the difference between self-loop input with output of itself, and the external input, i.e.

$$WIP_j(n+1) = WIP_j(n) + I_j(n) - O_j(n)$$

To link outputs to the inputs, we impose the following:

Assumption 1 (Flow conservation) The job shop system is mass conservative, that is for given flow probability matrix P we have

$$I_j(n) = \sum_{l=1}^{P} p_{lj} O_l(n)$$

for each workstation j.

We like to note that Assumption 1 is appropriate here for two reasons: First, loss of products within the job shop system can be included by modifying the flow probabilities between the workstations. And secondly, the assumption rules out dissipation, which similar to friction in a mechanical system eases the task of stabilizing a system. Hence, Assumption 1 represents the more difficult case and all following results also apply to the case with dissipation.

Here, we additionally focus on the operational layer only. As a consequence, we cannot determine the order release rates to any of the workstations.

Assumption 2 (Operational layer) The order release rates $i_{0j}(n)$ to each workstation j are determined externally and must therefore be considered as disturbances $d_j(n) := i_{0j}(n)$.

Utilizing Assumptions 1 and 2 allows us to simplify *WIP_i* to

$$WIP_{j}(n+1) = WIP_{j}(n) + I_{j}(n) - O_{j}(n)$$

= $WIP_{j}(n) + i_{0j}(n) + \sum_{l=1}^{p} p_{lj}O_{l}(n) - O_{j}(n)$ (1)

$$= WIP_j(n) + \sum_{\substack{l=1\\ l \neq j}}^p p_{lj}O_l(n) + (p_{jj} - 1) \cdot O_j(n) + d_j(n).$$

As we want to include RMTs into the workstations, we link system (1) to number of machine tools operating within the workstations. Note that from an economic point of view it only makes sense to buy new machinery if the current capacity is insufficient to deal with all orders. This typically leads to high *WIP* levels, which allow us to rewrite the output as

$$O_i(n) = n^{\text{DMT}} r^{\text{DMT}} + u(n) r^{\text{RMT}}$$
⁽²⁾

At low levels or other extreme operating conditions, however, different capacity adjustments may be required [26].

Our idea, which we follow in this paper, is to ensure fidelity of a feedback, i.e. to guarantee that all workstations operate close to predefined *WIP* levels. If this property can be shown for a feedback at hand, then the assumption of a high *WIP* level can be shown to hold.

More formally, the latter assumption reads:

Assumption 3 (High WIP level) At any time instant n, the WIP levels in all workstations are at least as high as the total machine capacity, i.e. (2) holds.

One way to prove that Assumption 3 holds is to show that (1) is asymptotically stable for a feedback at hand. To this end, let $x = (WIP_1, \ldots, WIP_p) \in \mathbb{X} \subset X$ represent the *WIP* level of all workstations and $u = (u_1, \ldots, u_p) \in \mathbb{U} \subset U$ denote the vector of RMTs assigned to all workstations. Here, the sets \mathbb{X} and \mathbb{U} allow us to incorporate possibly wanted constraints on the *WIP* level and the total number of RMTs. Utilizing Assumption 3, then from (1) we obtain

$$x(n+1) = x(n) + P \cdot \left(n^{\text{DMT}} r^{\text{DMT}} + u(n) r^{\text{RMT}}\right) + d(n)$$

=: $f(x(n), u(n), d(n)),$ (3)

see also [27] for further modelling details. Using the latter-notation, we can define the concept of asymptotic stability formally:

Definition 1 Suppose a system (3), a predefined reference value x^* and a control $u(\cdot)$ to be given such there exists a forward invariant set $Y \subset X$. If there exists function $\beta \in \mathcal{KL}$ such that

 $\|x(n) - x^*\| \le \beta(\|x_0 - x^*\|, n)$ (4)

holds for all $x_0 \in Y$ and all $n \in \mathbb{N}_0$, then the control $u(\cdot)$ asymptotically stabilizes x^* .

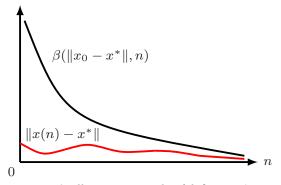


Fig. 2: Illustrate example of definition 1

Now we directly obtain the following.

Corollary 1 Consider system (1) and a predefined reference value x_j^* . If for the control $u(\cdot)$ a set $Y \subset X$ is forward invariant such that

$$y \ge n^{\text{DMT}} r^{\text{DMT}} + u(n) r^{\text{RMT}} \quad \forall y \in Y, \ n \in \mathbb{N}_0$$
(5)

holds, then Assumption 3 holds.

In practical terms, inequality (5) ensures that for any chosen time instant n the *WIP* level is high enough such that all machine tools within a workstation work at full capacity. Forward invariance in turn means the control u ensures, that the buffers of all workstations will be refilled such that full capacity utilization in the next time step is guaranteed.

We like to note that in practice perfect tracking (Definition 1) is almost surely impossible, but needs to be extended to practical stability, cf. [20, Chapter 2]. Apart from workers, who may interfere with the processes, the latter is due to two facts: For one, any reconfiguration requires time, which results in a time delayed process. And secondly, only an integer number of RMTs can be assigned to a workstation, whereas typical feedbacks consider convex sets. To deal with both issues in long term, we propose to consider MPC as a control scheme, which is able to explicitly handle such constraints.

To make the first step into this direction, in this paper, we consider control of the manufacturing process in the continuous optimization case and compare an MPC to the standard PID implementation.

3 MODEL PREDICTIVE CONTROL

To achieve the goal of asymptotic stability, we propose to utilize MPC. The idea of the latter is to approximate the solution of the infinite horizon optimal control problem with key performance index $\ell(\cdot, \cdot)$

$$J_{\infty}(x_0, u) = \sum_{k=0}^{\infty} \ell(x(k), u(k))$$
(6)

subject to the dynamics (3) and constraints $x \in X$, $u \in U$. Note that the direct integration of both the key performance index and of the constraints presents a major difference to a PID controller. While the latter needs to be tuned by an expert to adhere the constraints and perform well given an external index, no further action is required for the MPC controller.

Before we specify the MPC problem to our setting, we introduce the general background of the method. Following literature [20], we impose the following standard assumptions:

Assumption 4 For $x^* \in \mathbb{X}$ there exists $u^* \in \mathbb{U}$ such that $f(x^*, u^*, d) = x^*$.

Assumption 5 The stage cost $\ell : X \times U \to \mathbb{R}_{\geq 0}$ satisfies $\ell(x^*, u^*) = 0$ and $\ell(x, u) > 0$ for all $u \in \mathbb{U}$ if $x \neq x^*$.

The optimal value function corresponding to (6) is given by $V_{\infty}(x_0) = \inf_{u \in \mathbb{U}} J_{\infty}(x_0, u)$ and based on dynamic programming principles we obtain

$$V_{\infty}(x_0, d_0) = \inf_{u \in \mathbb{U}} \{ \ell(x_0, u) + V_{\infty}(f(x_0, u, d_0)) \}$$

and can derive an optimal feedback control law

$$\mu_{\infty}(x(n)) = \underset{u \in \mathbb{U}}{\operatorname{argmin}} \{\ell(x(n), u) + V_{\infty}(f(x(n), u, d(n)))\}$$

by using Bellmans optimality principle. Since this optimal control problem is typically computationally intractable, MPC approximates the respective solution via a three step procedure: After obtaining the current state of the system, a truncated optimal problem with finite prediction horizon is solved to obtain a corresponding optimal control sequence. Then, only the first element of this sequence is applied and the prediction horizon is shifted, which renders the method to be iteratively applicable. Then computationally complex part is the solution of the truncated problems

$$\min J_N(x_0, u, d) = \sum_{k=0}^{N-1} \ell(x(k), u(k))$$
(7)

subject to
$$x(k+1) = f(x(k), u(k), d(k)),$$

 $x(0) = x_0$
 $x(k) \in \mathbb{X} \ \forall k \in \{0, \dots, N\}$
 $u(k) \in \mathbb{U} \ \forall k \in \{0, \dots, N-1\}$
 $d(k) \in \mathbb{D} \ \forall k \in \{0, \dots, N-1\}$

required in the second step of Algorithm 1. For simplicity of exposition we assume that a minimizer $u^*(\cdot) := \operatorname{argmin}_{u \in \mathbb{U}} J_N(x_0, u, d)$ of (7) is unique. Combined, these steps reveal the following algorithm:

 Algorithm 1 Basic model predictive control method

 Input: $N \in \mathbb{N}$.

 1: for n = 0, ... do

 2: Measure current WIP levels x(n) and set $x_0 := x(n)$

 3: Compute control inputs u(n) by solving (7)

4: Apply $\mu_N(x(n)) = u^*(0)$ to all workstations

5: end for

Output: Feedback $\mu_N(\cdot)$

Applying Algorithm 1, for a given initial value $x_0 = x(0)$, we obtain the closed-loop solution

$$x(n+1) = f(x(n), \mu_N(x(n)), d(n)).$$
(8)

Note that optimality in each iterate is not sufficient to guarantee stability in the sense of Definition 1. Yet, we can utilize the optimal value function $V_N(x_0) = J_N(x_0, u^*(\cdot))$ to recapitulate the following from [20, Lemma 5.4, Theorem 5.13]:

Lemma 1 Consider the optimal control problem (7) with prediction horizon $N \in \mathbb{N}$ and the additional terminal condition $x(N) = x^*$ and suppose that Assumptions 4 and 5 hold. Then for each $N \ge 2$ and each $x \in \mathbb{X}_{N-1}$ we have $V_{N-1}(x) \ge V_N(x)$.

Theorem 1 Suppose the assumptions of Lemma 1 to hold. If there exist functions α_1 , α_2 and $\alpha_3 \in \mathcal{K}_{\infty}$ such that

$$\alpha_1(\|x - x^*\|) \le V_N(x) \le \alpha_2(\|x - x^*\|)$$
(9)

$$\alpha_3(\|x - x^*\|) \le \inf_{u \in \mathbb{H}} \ell(x, u)$$
(10)

holds, then μ_N is asymptotically stabilizing the closedloop (8) in the sense of Definition 1.

Theorem 1 provides the general background for our task of asymptotically stabilizing a job shop system with RMTs. Hence, our aim now is to prove that the method applies to our case. The particular difficulty with MPC is that a suitable prediction horizon N is typically unknown, yet if N reveals a stabilizing control, then also the feedback with N + j for $j \in \mathbb{N}$ stabilizes the closedloop [28]. Here, we show that for N = 2, the solution of problem (7) can be computed explicitly without the requirement of an optimization routine. Therefore, also all feedbacks with $N \ge 2$ asymptotically stabilize the closed-loop, which renders the method to be applicable in general.

For technical reasons, we require

Assumption 6 The flow probability matrix P between the workstations is invertible.

Then we can utilize our dynamics (3) together with Assumption 6 to show the following:

Proposition 1 *Consider problem (7) for the job shop system (3) together with the stage costs*

$$\ell(x(k), u(k)) = \|x(k) - x^*\|_2^2 + \lambda \cdot \|u(k) - u^*\|_2^2$$
(11)

for some predefined desired equilibrium (x^* , u^*). Then Theorem 1 holds for N = 2 with

$$\alpha_1(s) = \alpha_3(s) = s^2$$

$$\alpha_2(s) = (1 + \lambda \|\theta_1\|_2^2 + \|\theta_2\|_2^2 + \lambda \|\theta_3\|_2^2)s^2$$

where

$$\theta_1 = (4\lambda \cdot \mathbf{Id} + 2a_1^{\mathsf{T}}a_1)^{-1}(-2a_1^{\mathsf{T}} + \frac{-P^{-1}}{r^{\mathsf{RMT}}} \cdot 2\lambda)$$

$$\theta_2 = \mathbf{Id} + r^{\mathsf{RMT}}P\theta_1$$

$$\theta_3 = \frac{P^{-1} + r^{\mathsf{RMT}}\theta_1}{r^{\mathsf{RMT}}}.$$

Moreover, the stabilizing MPC feedback is given by $\mu_2(x) = \theta_1(x - x^*) + u^*$.

The details of proof are given in the appendix.

In practice, the stage cost (11) may represent any performance indicator or a scalarized combination of several indicators. Hence, it is possible to model maximization of throughput, profit or quality as well as minimizing lead time, energy requirements or costs directly.

Given the result from Proposition 1, MPC is as readily available for the job shop system problem including RMTs as PID. A particular conclusion from this result is that, upon implementation, one does not have to worry about stability of the closedloop when choosing the prediction horizon length as stability comes for free. Hence, similar to PID, no expert knowledge is required to control the system. In contrast to stability, however, PID requires internal knowledge of the key performance indexes used to evaluate the feedback as well as good command of how to appropriately adapt the PID parameters to perform well for these indexes. The latter task becomes even more difficult if connected PID controllers, e.g., one per workstation, need to be considered and to be adjusted simultaneously. In such a MIMO case, one may apply optimization mehtods, e.g. particle swarm optimization [29] or iterated linear matrix inequalities [30]. For MPC, no further knowledge and no adaptation phase is required as KPIs can be used as cost criterion and are therefore optimized by design.

Remark 1 Due to the additional terminal endpoint constraints, recursive feasibility is guaranteed automatically, i.e. if the initial state of the job shop system adheres all constraints, then there always exists a solution to problem (7). [20] and the MPC procedure can be applied without running into a dead end.

Remark 2 While the solution derived from optimization typically outperforms the decision based on worker experience, the computational cost grows with the dimension of the system and may be intractable, i.e. the best solution may not found in a reasonable time. Within a job shop system, this may

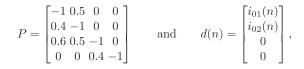
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be the case if the initial state of the job shop system is far from the desired equilibrium. In this case, decentralized or distributed control could be applied for the high order system [31, 32], which are out of the scope of this article.

To complement and check our theoretical findings, we next consider a numerical example to illustrate our results.

4 CASE STUDY

Within this section, we considered the multi workstation system sketched in Fig. 3. Since products can be manufactured cost efficiently with a high productivity by means of dedicated machines, and diversity of customized product at a low quantity effectively via reconfigurable machines, we included both of RMTs and DMTs for all workstations. This kind of combination and co-existence in industrial practice can be observed frequently [33] and naturally occurs when a new type of machine tool is introduced. The respective dynamics are given by (3) with parameters and initial values according to Tab. 2 as well as flow matrix and external input rates



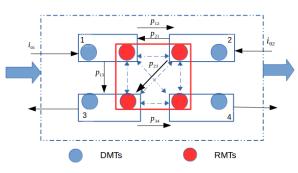


Fig. 3: Multi workstation multi product job shop system

Table 2: The variables definition
in the job shop system

Variable	Description
$x(0) = [40 \ 40 \ 40 \ 30]^{\top}$	Work in process (WIP) level
$r^{\rm DMT} = 3$	Production rate of DMT
$r^{\mathrm{RMT}} = 2$	Production rate of RMT
$n^{\rm DMT} = [5 \ 4 \ 5 \ 2]^\top$	Number of DMTs for each WS
$u = [2 \ 1 \ 2 \ 1]^{\top}$	Number of RMTs for each WS
m = 6	Maximum value of RMTs
$x^* = [25, 22, 25, 16]^\top$	Planned work in process (WIP)

where the latter include a demand fluctuation in a certain period, which is modeled by a sin function

$$i_{01}(n) = \begin{cases} 10+3|\sin(0.1\pi n)|, & 20 < n \le 30\\ 10, & \text{else} \end{cases}$$
$$i_{02} = 6. \tag{12}$$

Then, our goal was to steer the WIP level of each workstation to the respective desired value x_j^* while considering the state and control constraints

$$x(N) = x^*, \quad 0 \le u_j(n) \quad \text{and} \quad \sum_{j=1}^4 u_j(n) \le m$$
 (13)

We imposed the stage cost function (11), which satisfies Assumption 5. From Assumption 4, we then obtained

$$u^* = \frac{-P^{-1} \cdot d - \mathbf{1} \cdot n^{\text{DMT}} \cdot r^{\text{DMT}}}{r^{\text{RMT}}}.$$
(14)

In order to increase the basin of attraction, we chose N = 16 and obtained the simulation results sketched in Figs. 4 and 5. As benchmark, we complemented these figures by respective graphs using a PID controller. As PID does not allow to include constraints on the total number of RMTs, we additionally imposed the truncation

$$u_{j}(n) = \begin{cases} u_{j}(n), & \text{if } \sum_{j=1}^{p} u_{j}(n) \leq m \\ \frac{m}{\sum_{j=1}^{p} u_{j}(n)}, & \text{else.} \end{cases}$$
(15)

such that PID always adheres to this constraint. The parameters of the PID controller were tuned manually by extensive simulations to reduce oscillations as

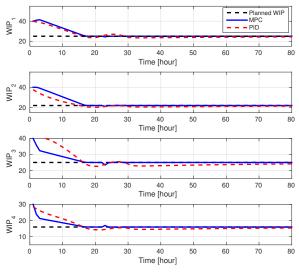


Fig. 4: WIP level for MPC and PID

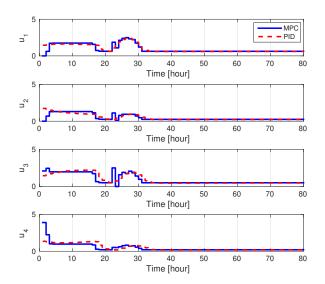


Fig. 5: Assigned RMTs by MPC and PID

much as possible but without external optimization technique. As simulations have shown that the D component shows no further improvement, we chose a PI controller with $k_p = 0.5$, $k_i = 0.01$ and $k_d = 0$.

As expected, in Fig. 4 we observe that the proposed method is capable of tracking the desired WIP value for each workstation. The allocation of RMTs is displayed in Fig. 5.

From Fig. 5, we observed that for both MPC and PID, the number of RMTs assigned to particular workstation is identical from time instant n = 34 onwards. Comparing this to the WIP levels in Fig. 4, we found that MPC is tracking the desired values x_j^* perfectly for all workstations j = 1, ..., 4. PID, on the other hand, shows an offset, which we were unable to solve despite extensive tuning of the control parameters.

For $n \in [0, 33]$ in Fig. 5, we observed that both controllers result in a very different assignment of RMTs, which however is not well reflected in standard comparison errors. Here, we considered integral absolute error (IAE), integral square error (ISE), integral absolute control (IAU) and integral square control (ISU) to analyze the results displayed in Fig. 4 and 5, cf. Tab. 3 for details. Considering the differences in the assignments, time instants n = 0 and n = 22were of particular interest: At n = 0, PID chose almost identical assignments of RMTs for all workstations, whereas MPC moved RMTs to workstations 3 and 4 only. As a result, the WIP levels for workstations 3 and 4 in the MPC case dropped significantly faster than for PID, whereas WIP levels for workstations 1 and 2 were only slightly higher, cf. Fig. 4. At n = 22, both PID and MPC reacted to the change of the input rate. The following curve was almost identical for both controllers and approximates a sin function, which was to be expected given the nature of the input rate change. In contrast to PID, however, MPC started with a very strong peak in workstations 1, 2 and 3.

			unu 1111 ·	C		
		IAE	IAU	ISE	ISU	
	WS1	199.40	77.41	1496.40	97.02	
PID	WS2	161.06	40.25	1032.42	37.80	
	254.92	74.51	2245.99	104.74		

37.10

76.83

39.73

73.47

36.38

866.20

1882.57

2163.74

622.92

400.50

31.85

100.05

33.81

103.40

39.38

F

MPC

WS4

WS1

WS2

WS3

WS4

162.59

155.24

165.45

83.10

62.36

To further test the proposed method, we additionally simulated cases for different initial values. In the context of MPC with terminal conditions, initial values far from the desired equilibrium require a large prediction horizon N, which in turn may cause problems in solving problem (7). To test the job shop system problem, we considered the cases 1) x(0) = [30]30 30 20]^T, 2) $x(0) = [40 30 20 30]^{T}$, 3) $x(0) = [40 40]^{T}$ 40 30]^T, and 4) $x(0) = [50 \ 40 \ 50 \ 40]^{T}$, for which the resulting trajectories are displayed in Fig. 6. We like to note that this cannot be regarded as a complete test, which would require to check for control forward invariance of a set of initial conditions. Yet, the cases already assume quite excessive deviations from the desired point of operation and from a practical point of view, such occasions are already rather unlikely. Still, we observed that in all cases the trajectories converged to the desired values x_i^* for all workstations j = 1, ..., 4. Additionally, we like to highlight a peculiarity arising for workstations 2 and 4 at time instant n = 23, when the trajectories were almost at x_i^* , cf. the magnified section in Fig. 6. Here, the trajectories showed an almost inverse behavior. In fact, MPC chose to cause a deviation from x_i^* on purpose as it recognized that this is necessary to steer the trajectory exactly to x_i^* at later time instances.

Remark 3 We additionally like to note that the terminal condition $x(N) = x^*$ has to reachable within the prediction horizon. Hence, depending on the range of initial conditions and of the external input rate d, which the user wants to allow, the prediction horizon must be chosen large enough. If the input rate and initial conditions are contained in a region for which a feasible solution exists, then Proposition 1 together with Theorem 1 guarantee that MPC asymptotically stabilizes the job shop system.

Table 3: Comparison between standardPID and MPC

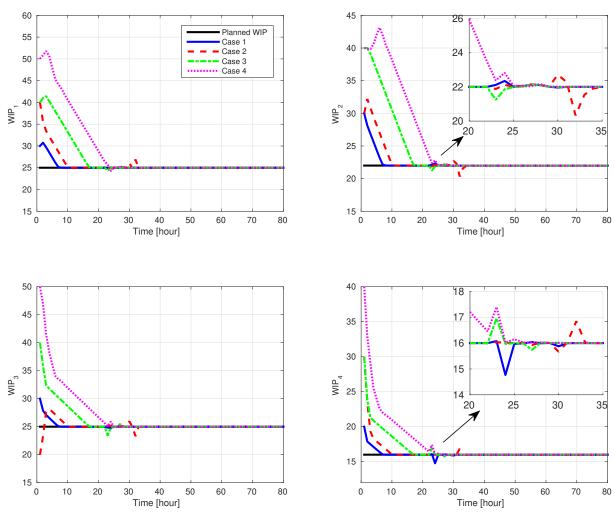


Fig. 6: WIP levels for different initial conditions

Last, we considered the case of a dynamic flow, which includes the product mix handled by the workstations and which leads to the modification of off-diagonal values of P given by

$$p_{21}(n, O_1(u(n)), d(n)) := \frac{i_{02}(n)}{p_{12}O_1(n) + i_{02}(n)}$$

$$p_{12}(n, O_2(u(n)), d(n)) := \frac{i_{01}(n)}{p_{21}O_2(n) + i_{01}(n)}$$

$$p_{23}(n, O_1(u(n)), d(n)) := \frac{p_{12}O_1(n)}{p_{12}O_1(n) + i_{02}(n)}$$

$$p_{13}(n, O_2(u(n)), d(n)) := \frac{p_{21}O_2(n)}{p_{21}O_2(n) + i_{01}(n)}$$

$$p_{34}(n, O_2(u(n)), d(n)) := \frac{p_{23}O_2(n)}{p_{23}O_2(n) + p_{13}O_1(n)}$$

Respective results are illustrated in Fig. 7. Here, we observed that – despite availability of all information regarding workstation outputs – the resulting trajectories showed oscillations. Therefore, the closed-loop system was not asymptotically stabilized but showed practical asymptotic stability for the chosen prediction horizon N = 16 only. For a complete stability proof in the case

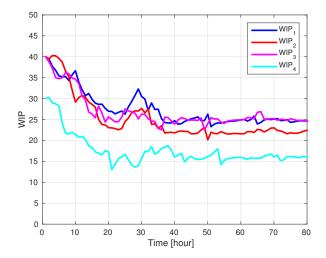


Fig. 7: Variations of WIP in the dynamic flow

of time varying dynamics, also time varying Lyapunov arguments would be required, which was out of the scope of this article. Yet the simulation indicated that the applicability of MPC in the time varying case is not perfectly straight forward and requires further analysis.

Based on these results, we conjecture that the difference between PID and MPC seen in Figs. 4 and 5 at n = 0 could be reduced significantly by choosing different PID control parameters for each workstation. As this is unnecessary for MPC, we conclude that from a practitioners point of view MPC is more easily accessible. Regarding n = 22 and the deviation from the desired values observed for PID, MPC – at least for this example setting – also shows improved performance. Additionally, we conjecture that this plug and play property will allow us to straight forward integrate the integer constraints and reconfiguration delays mentioned in Section 2, whereas for PID these may cause serious problems in the stability proof.

5 CONCLUSION AND OUTLOOK

Reconfigurability is a key enabler for handling exceptions and performance deteriorations in manufacturing operations. In the context of changing capacity requirements, reconfigurable machine tools (RMTs) offer a machinery-based alternative to labororiented methods, which are already established in practice. Combined with suitable planning and control methods, RMTs may become a powerful enabler of Industry 4.0 concepts.

In this paper, we showed that RMTs allow to adjust capacity and functionality of a job shop system effectively in the presence of demand fluctuations. To this end, we considered the WIP levels for each workstation, which we controlled by optimally reallocating RMTs using MPC. Utilizing equilibrium terminal conditions, we showed that asymptotic stability of the closed-loop system can be guaranteed for the job shop system case with RMTs regardless of the chose prediction horizon, which is typically hard to obtain for applications. As a consequence, we were able to show that the choice of the prediction horizon solely depends on the operating range of the job shop system, which in practice is defined by responsible managers. Hence, MPC represents a readily available and plug-and-play applicable tools to include RMTs into job shop systems.

To illustrate the latter, we presented a numerical case study of a four workstation two product job shop. We compared the applicability of MPC with standard PID and observed that applying MPC directly showed better results as compared to PID, which was tuned manually, which showed the plug-and-play property of MPC.

In the future, we will extend the model to incorporate reconfiguration delays and transportation times as well as integer programming methods for the assignment of RMTs. Here, we expect even better results for MPC, which allows to directly address these issues. Moreover, stability without terminal conditions is also of interest, especially as the integer constraints may lead to large combinatorial problems which may be reduced drastically if the terminal conditions can be dropped. Moreover, we will move to adherence of delivery dates by modifying the cost functional and dynamics respectively.

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APPENDICES

Proof of Proposition 1:

Given the running cost (11), $\alpha_1(s) = \alpha_3(s) = s^2$ satisfy (9) and (10). Hence, only the bound $\alpha_2(||x - x^*||) \ge V_N(x)$ needs to be established. Based on Lemma 1, we conclude that if $V_2(x) \le \alpha_2(||x - x^*||)$ holds, then $V_N(x) \le \alpha_2(||x - x^*||)$ holds for any $N \ge 2$. Based on the dynamic programming principle, we get $V_2(x) = \ell(x, \mu_2(x)) + V_1(f(x, \mu_2(x)))$ and therefore

$$\mu_{2}(x) = \underset{u \in \mathbb{U}}{\operatorname{argmin}} (\ell(x, u) + V_{1}(f(x, u)))$$

=
$$\underset{u \in \mathbb{U}}{\operatorname{argmin}} \|x - x^{*}\|_{2}^{2} + \lambda \cdot \|u - u^{*}\|_{2}^{2}$$

+
$$\|(x - x^{*} + P \cdot n^{\text{DMT}}r^{\text{DMT}}) + d + r^{\text{RMT}} \cdot P \cdot u\|_{2}^{2}$$

+
$$\lambda \cdot \| - \frac{P^{-1}(x - x^{*} + P \cdot n^{\text{DMT}}r^{\text{DMT}} + d)}{r^{\text{RMT}}} - u\|_{2}^{2}$$

Next, we set

$$a_1 := r^{\text{RMT}} \cdot P$$

$$a_2 := x - x^* + P \cdot n^{\text{DMT}} r^{\text{DMT}} + d$$

$$a_3 := -\frac{P^{-1}(x - x^* + P \cdot n^{\text{DMT}} r^{\text{DMT}} + d)}{r^{\text{RMT}}}$$

and obtain

$$\mu_2(x) = \operatorname{argmin}(\|x - x^*\|_2^2 + \lambda \cdot \|u - u^*\|_2^2 + \|a_2 + a_1 \cdot u\|_2^2 + \lambda \cdot \|a_3 - u\|_2^2.$$

Hence, we have

$$\frac{\partial \mu_2(x)}{\partial u} = 2\lambda(u - u^*) + 2a_1^\top (a_2 + a_1 \cdot u) - 2\lambda(a_3 - u)$$
$$= (4\lambda \cdot \mathbf{Id} + 2a_1^\top a_1)u + 2a_1^\top a_2 - 2\lambda(a_3 + u^*)$$

Since
$$a_3 = -\frac{P^{-1}(x-x^*)}{r^{\text{RMT}}} + u^*$$
 then

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$$u = (4\lambda \cdot \mathbf{Id} + 2a_{1}^{\top}a_{1})^{-1} (\overbrace{\frac{\partial \mu_{2}(x)}{\partial u}}^{0} - 2a_{1}^{\top}a_{2} + 2\lambda(a_{3} + u^{*}))$$

$$= (4\lambda \cdot \mathbf{Id} + 2a_{1}^{\top}a_{1})^{-1} (-2a_{1}^{\top}a_{2} + 2\lambda(-\frac{P^{-1}(x - x^{*})}{r^{\mathrm{RMT}}} + 2u^{*}))$$

$$= \overbrace{(4\lambda \cdot \mathbf{Id} + 2a_{1}^{\top}a_{1})^{-1}(-2a_{1}^{\top} + \frac{-P^{-1}}{r^{\mathrm{RMT}}} \cdot 2\lambda)(x - x^{*}) + (4\lambda \cdot \mathbf{Id} + 2a_{1}^{\top}a_{1})^{-1}(4\lambda u^{*} - 2a_{1}^{\top}(P \cdot n^{\mathrm{DMT}}r^{\mathrm{DMT}})))$$

$$= \theta_{1}(x - x^{*}) + u^{*}$$

As the problem is convex, u is the unique optimal solution according to the MPC scheme, which in turn allows us to set $\mu_2 = u$.

 $\begin{aligned} & \text{Utilizing the closed-loop} \\ f(x, \mu_2(x)) &= x + P n^{\text{DMT}} r^{\text{DMT}} + \\ d + r^{\text{RMT}} P(\theta_1(x - x^*) + u^*), \text{ we obtain} \\ V_1(f(x, \mu_2(x)) &= \| (\mathbf{Id} + r^{\text{RMT}} P \theta_1)(x - x^*) + (P n^{\text{DMT}} \\ r^{\text{DMT}} + d + r^{\text{RMT}} P u^*) \|_2^2 + \lambda \cdot \| \frac{(P^{-1} + r^{\text{RMT}} \theta_1)(x - x^*)}{r^{\text{RMT}}} \\ &+ \frac{n^{\text{DMT}} r^{\text{DMT}} + P^{-1} d + r^{\text{RMT}} u^*}{r^{\text{RMT}}} \|_2^2. \end{aligned}$ (16)

Last, we substitute (14) into (16), which then reads

$$V_{2}(x) = \ell(x, \mu_{2}(x)) + V_{1}(f(x, \mu_{2}(x)))$$

$$= \|x - x^{*}\|_{2}^{2} + \lambda \|\theta_{1}(x - x^{*})\|_{2}^{2}$$

$$+ \|\overbrace{(\mathbf{Id} + r^{\mathrm{RMT}}P\theta_{1})}^{\theta_{2}}(x - x^{*})\|_{2}^{2}$$

$$+ \lambda \cdot \|\overbrace{(P^{-1} + r^{\mathrm{RMT}}\theta_{1})}^{\theta_{3}}(x - x^{*})\|_{2}^{2}$$

$$\leq \|x - x^{*}\|_{2}^{2} + \lambda \|\theta_{1}\|_{2}^{2}\|x - x^{*}\|_{2}^{2} + \|\theta_{2}\|_{2}^{2}\|x - x^{*}\|_{2}^{2}$$

$$+ \lambda \|\theta_{3}\|_{2}^{2}\|x - x^{*}\|_{2}^{2}$$

$$= (1 + \lambda \|\theta_{1}\|_{2}^{2} + \|\theta_{2}\|_{2}^{2} + \lambda \|\theta_{3}\|_{2}^{2})\|x - x^{*}\|_{2}^{2}$$

$$(17)$$

$$= (1 + \lambda \|\theta_{1}\|_{2}^{2} + \|\theta_{2}\|_{2}^{2} + \lambda \|\theta_{3}\|_{2}^{2})\|x - x^{*}\|_{2}^{2}$$

Therefore, we may define the bound $\alpha_2(s) = (1 + \lambda \|\theta_1\|_2^2 + \|\theta_2\|_2^2 + \lambda \|\theta_3\|_2^2)s^2.$

Hence, $V_2(x)$ is a Lyapunov function and the assumptions of Theorem 1 hold. Accordingly, the assertion follows and $\mu_2(x) = \theta_1(x - x^*) + u^*$ asymptotically stabilizes the closed-loop system.

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